The Classical Monetary Model

(Stochastic Version; From “Economic Growth:Theory and Numerical Solution Methods”, by A. Novales, E, Fernández, y J. Ruiz, 2009, Springer Verlag)

The representative agent solves the problem:

$$\underset{\{c_t, M_{t+1}, k_{t+1}, V_{t+1}\}_{t=0}^{\infty}}{Max} E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, M_{t+1}/P_t)$$

subject to:

$$(1+\tau^c)c_t + k_{t+1} - (1-\delta)k_t + \frac{M_{t+1}}{P_t} + \left[\frac{V_{t+1}}{(1+i_t)P_t}\right] = (1-\tau^y)A_t k_t^{\alpha} + \frac{M_t}{P_t} + \frac{V_t}{P_t} + \gamma_t,$$

given k_0, M_0, V_0 . The productivity shock is assumed to obey the stochastic process

$$\ln A_t = (1-\rho_A)\ln A_{ss} + \rho_A A_{t-1} + \varepsilon_{A,t}, \ \ |\rho_A| < 1, \ \varepsilon_{A,t} \underset{iid}{\sim} N(0, \sigma_A^2).$$

The government raises income and consumption taxes, prints money and issues nominal bonds, which are bought at price $1/(1+i_t)$. It also provides transfers γ_t to the private sector. The government budget constraint is,

$$\tau^c c_t + \tau^y A_t k_t^\alpha + \frac{M_{t+1} - M_t}{P_t} + \left[\frac{V_{t+1}}{(1+i_t)P_t} - \frac{V_t}{P_t}\right] = \gamma_t.$$

The fiscal authority chooses sequences $\{\tau^c, \tau^y\}$ and $\{\gamma_t\}_{t=0}^{\infty}$ .To guarantee stability of the public debt trajectory, we will assume that:

$$\gamma_t = \bar{\gamma} - \eta \frac{V_t}{P_t}, \tag{6.147}$$

which implies,

$$\tilde{b}_{t+1} = (1+i_t)\left[\gamma - \tau^c c_t - \tau^y A_t k_t^\alpha - \tilde{m}_{t+1} + \frac{\tilde{m}_t}{1+\pi_t}\right] + (1-\eta)\frac{1+i_t}{1+\pi_t}\tilde{b}_t. \tag{6.148}$$

where, as in previous sections, $\tilde{b}_{t+1} = \frac{V_{t+1}}{P_t}, \tilde{m}_{t+1} = \frac{M_{t+1}}{P_t}$. Then, $\tilde{b}_{t+1}$ will be stable so long as $1 > \eta > \frac{r_{ss}}{1+r_{ss}}$, where: $1 + r_{ss} = (1+i_{ss})/(1+\pi_{ss})$.

Additionally, we will assume that the monetary authority chooses the sequence of nominal interest rates $\{i_t\}_{t=0}^{\infty}$ according to a Taylor's rule:

$$\hat{\imath}_t = \rho_i \hat{\imath}_{t-1} + \rho_\pi \hat{\pi}_t + \rho_y \hat{y}_t + \varepsilon_{i,t}, \ \ |\rho_i| < 1, \ \varepsilon_{i,t} \underset{iid}{\sim} N(0, \sigma_i^2),$$

$$\text{where } \hat{\imath}_t \equiv \ln\left(\frac{1+i_t}{1+i_{ss}}\right); \ \hat{\pi}_t \equiv \ln\left(\frac{1+\pi_t}{1+\pi_{ss}}\right); \ \hat{y}_t \equiv \ln\left(\frac{y_t}{y_{ss}}\right) \underbrace{=}_{y_t = A_t k_t^\alpha} \hat{A}_t + \alpha \hat{k}_t,$$

Case 1: $\rho_\pi = 0$.

If $\rho_\pi = 0$, the Taylor rule becomes:

$$\hat{\imath}_t = \rho_i \hat{\imath}_{t-1} + \rho_y \left(\hat{A}_t + \alpha \hat{k}_t\right) + \varepsilon_{i,t}, \tag{6.149}$$

The Lagrangian of the representative agent is,

$$L = E_0 \left[\sum_{t=0}^{\infty} \beta^t \left\{ U(c_t, M_{t+1}/P_t) + \lambda_t \left(\begin{array}{c} (1-\tau^y) A_t k_t^{\alpha} + \frac{M_t}{P_t} + \frac{V_t}{P_t} + \gamma_t - (1+\tau^c) c_t - k_{t+1} + \\ (1-\delta) k_t - \frac{M_{t+1}}{P_t} - \left[\frac{V_t}{(1+i_t) P_t} \right] \end{array} \right) \right\} \right]$$

with first order conditions:

$$\begin{aligned}
U_c(c_t, \tilde{m}_{t+1}) &= (1+\tau^c)\lambda_t, \\
\lambda_t &= \beta E_t \left[\lambda_{t+1} \left((1-\tau^y) \alpha A_{t+1} k_{t+1}^{\alpha-1} + 1 - \delta \right) \right], \\
-U_m(c_t, \tilde{m}_{t+1}) \frac{1}{P_t} + \lambda_t \frac{1}{P_t} &= \beta E_t \left(\lambda_{t+1} \frac{1}{P_{t+1}} \right), \\
\lambda_t \frac{1}{P_t(1+i_t)} &= \beta E_t \left(\lambda_{t+1} \frac{1}{P_{t+1}} \right),
\end{aligned}$$

where $\tilde{m}_{t+1} = M_{t+1}/P_t$.

Plugging the first optimality condition into the other three conditions, we get:

$$U_c(c_t, \tilde{m}_{t+1}) = \beta E_t \left[U_c(c_{t+1}, \tilde{m}_{t+2}) \left((1-\tau^y)\alpha A_{t+1} k_{t+1}^{\alpha-1} + 1 - \delta \right) \right]. \tag{6.150}$$

$$-U_m(c_t, \tilde{m}_{t+1}) + \frac{U_c(c_t, \tilde{m}_{t+1})}{(1+\tau^c)} = \beta E_t \left[\frac{U_c(c_{t+1}, \tilde{m}_{t+2})}{(1+\tau^c)} \frac{1}{1+\pi_{t+1}} \right]. \tag{6.151}$$

$$U_c(c_t, \tilde{m}_{t+1}) = \beta(1+i_t) E_t \left[U_c(c_{t+1}, \tilde{m}_{t+2}) \frac{1}{1+\pi_{t+1}} \right]. \tag{6.152}$$

From (6.151) and (6.152) we get:

$$\frac{i_t}{1+i_t} = (1+\tau^c)\frac{U_m(c_t, \tilde{m}_{t+1})}{U_c(c_t, \tilde{m}_{t+1})}. \tag{6.153}$$

For the utility function $U(c_t, M_{t+1}/P_t) = \frac{\left[c_t (M_{t+1}/P_t)^{\theta}\right]^{1-\sigma} - 1}{1-\sigma}$, $\sigma > 0$, (6.153) becomes:

$$\frac{i_t}{1+i_t} = \theta(1+\tau^c)\frac{c_t}{\tilde{m}_{t+1}}. \tag{6.154}$$

From the budget constraints for the representative agent and the government, we obtain the global constraint of resources in the economy:

$$c_t + k_{t+1} - (1-\delta)k_t = A_t k_t^{\alpha}. \tag{6.155}$$

Let us now recollect the set of optimality conditions that we are going to log-linearize in order to solve the model under the assumed utility function. From (6.150), (6.152), (6.154), (6.155), we get,

$$c_t^{-\sigma}\tilde{m}_{t+1}^{\theta(1-\sigma)} = \beta E_t\left[c_{t+1}^{-\sigma}\tilde{m}_{t+2}^{\theta(1-\sigma)}\left((1-\tau^y)\alpha A_{t+1}k_{t+1}^{\alpha-1}+1-\delta\right)\right], \quad (6.156)$$

$$c_t^{-\sigma}\tilde{m}_{t+1}^{\theta(1-\sigma)} = \beta(1+i_t)E_t\left[c_{t+1}^{-\sigma}\tilde{m}_{t+2}^{\theta(1-\sigma)}\frac{1}{1+\pi_{t+1}}\right], \quad (6.157)$$

$$\theta(1+\tau^c)\frac{c_t}{\tilde{m}_{t+1}} + \frac{1}{1+i_t} - 1 = 0, \quad (6.158)$$

$$c_t + k_{t+1} - (1-\delta)k_t = A_t k_t^{\alpha}. \quad (6.159)$$

Steady-state

From (6.156), (6.157), (6.158), (6.159), we get:

$$k_{ss} = \left[\frac{(1-\tau^y)\alpha A_{ss}}{\frac{1}{\beta} - (1-\delta)} \right]^{\frac{1}{1-\alpha}}.$$

$$c_{ss} = A_{ss} k_{ss}^{\alpha} - \delta k_{ss}.$$

$$\pi_{ss} = \beta(1 + i_{ss}) - 1.$$

$$m_{ss} = \theta(1+\tau^c) c_{ss} \frac{1 + i_{ss}}{i_{ss}}.$$

From (6.148) we get b_{ss}, while γ_{ss} is obtained from (6.147).

Log-linearization

To log-linearize this system (6.156)-(6.159), we rewrite the system as,

$$e^{-\sigma(\ln c_t)}e^{\theta(1-\sigma)(\ln \tilde{m}_{t+1})} = \beta E_t\left[e^{-\sigma(\ln c_{t+1})}e^{\theta(1-\sigma)(\ln \tilde{m}_{t+2})}\left((1-\tau^y)\alpha e^{\ln A_{t+1}}e^{(\alpha-1)(\ln k_{t+1})}+1-\delta\right)\right], \tag{6.160}$$

$$e^{-\sigma(\ln c_t)}e^{\theta(1-\sigma)(\ln \tilde{m}_{t+1})} = \beta e^{\tilde{i}_t}E_t\left[e^{-\sigma(\ln c_{t+1})}e^{\theta(1-\sigma)(\ln \tilde{m}_{t+2})}e^{-\ln \pi_{t+1}}\right], \tag{6.161}$$

$$\theta(1+\tau^c)e^{\ln c_t - \ln \tilde{m}_{t+1}} + e^{-\ln(1+i_t)} - 1 = 0, \tag{6.162}$$

$$e^{\ln c_t} + e^{\ln k_{t+1}} - (1-\delta)e^{\ln k_t} = e^{\tilde{A}_t}e^{\alpha(\ln k_t)}, \tag{6.163}$$

Let us denote the deviations from steady-state by: $\hat{u}_t = \ln(u_t/u_{ss})$, $u = c, k, \tilde{m}, A, 1+i, 1+\pi$. Using the fact that $E_t\hat{A}_{t+1} = \rho_A\hat{A}_t$, and also that $(1-\tau^y)\alpha A_{ss}k_{ss}^{\alpha-1} = \frac{1}{\beta} - (1-\delta)$, we obtain from (6.160)-(6.163),

$$0 = \frac{\sigma}{\beta}\hat{c}_t - \frac{\theta(1-\sigma)}{\beta}\widehat{\widetilde{m}}_{t+1} - \frac{\sigma}{\beta}E_t\hat{c}_{t+1} + \frac{\theta(1-\sigma)}{\beta}E_t\widehat{\widetilde{m}}_{t+2} + \left(\frac{1}{\beta}-(1-\delta)\right)\rho_A\hat{A}_t + \left(\frac{1}{\beta}-(1-\delta)\right)(\alpha-1)\hat{k}_{t+1}. \quad (6.164)$$

$$0 = \frac{\sigma}{\beta}\hat{c}_t - \frac{\theta(1-\sigma)}{\beta}\widehat{\widetilde{m}}_{t+1} + \frac{1+i_{ss}}{1+\pi_{ss}}\hat{\imath}_t - \frac{\sigma}{\beta}E_t\hat{c}_{t+1} + \frac{\theta(1-\sigma)}{\beta}E_t\widehat{\widetilde{m}}_{t+2} - \frac{1}{\beta}E_t\hat{\pi}_{t+1}. \quad (6.165)$$

$$\widehat{\widetilde{m}}_{t+1} = \hat{c}_t - \frac{1}{i_{ss}}\hat{\imath}_t \quad (6.166)$$

$$A_{ss}k_{ss}^{\alpha}\hat{A}_t + (A_{ss}k_{ss}^{\alpha} + (1-\delta)k_{ss})\hat{k}_t - c_{ss}\hat{c}_t - k_{ss}\hat{k}_{t+1} = 0 \quad (6.167)$$

where $\widehat{\widetilde{m}}_{t+1}$ is known at time t.

Plugging (6.166) into (6.164) and (6.165), we get:

$$\begin{aligned}0 \;=\;& \frac{\sigma-\theta(1-\sigma)}{\beta}\hat{c}_t + \frac{\theta(1-\sigma)}{\beta i_{ss}}\hat{\imath}_t - \frac{\sigma-\theta(1-\sigma)}{\beta}E_t\hat{c}_{t+1} - \frac{\theta(1-\sigma)}{\beta i_{ss}}E_t\hat{\imath}_{t+1} + \\ & \left(\frac{1}{\beta}-(1-\delta)\right)\rho_A\hat{A}_t + \left(\frac{1}{\beta}-(1-\delta)\right)(\alpha-1)\hat{k}_{t+1}, \end{aligned} \tag{6.168}$$

$$\begin{aligned}0 \;=\;& \frac{\sigma-\theta(1-\sigma)}{\beta}\hat{c}_t + \left(\frac{\theta(1-\sigma)}{\beta i_{ss}} + \frac{1+i_{ss}}{1+\pi_{ss}}\right)\hat{\imath}_t - \frac{\sigma-\theta(1-\sigma)}{\beta}E_t\hat{c}_{t+1} - \\ & \frac{\theta(1-\sigma)}{\beta i_{ss}}E_t\hat{\imath}_{t+1} - \frac{1}{\beta}E_t\hat{\pi}_{t+1}. \end{aligned} \tag{6.169}$$

We will now use the result, shown in Appendix 1, that in a log-linear approximation,

$$E_t\hat{\pi}_{t+1} = \hat{\imath}_t - \hat{r}_t \tag{6.170}$$

and also that:

$$\hat{r}_t = \frac{1}{1+r_{ss}}(1-\tau^y)\alpha A_{ss}k_{ss}^{\alpha-1}\left[\rho_A\hat{A}_t + (\alpha-1)\hat{k}_{t+1}\right] \tag{6.171}$$

From (6.149):

$$E_t\hat{\imath}_{t+1} = \rho_i\hat{\imath}_t + \rho_y\left(\rho_A\hat{A}_t + \alpha\hat{k}_{t+1}\right). \tag{6.172}$$

Using (6.170)-(6.172), we get from (6.168) :

$$
\begin{aligned}
0 \quad = \quad & \frac{\sigma-\theta(1-\sigma)}{\beta}\hat{c}_t + \frac{\theta(1-\sigma)}{\beta i_{ss}}(1-\rho_i)\hat{\imath}_t - \frac{\sigma-\theta(1-\sigma)}{\beta}E_t\hat{c}_{t+1} + \\
& \left[\left(\frac{1}{\beta}-(1-\delta)\right) - \frac{\theta(1-\sigma)}{\beta i_{ss}}\rho_y\right]\rho_A\hat{A}_t - \qquad (6.173) \\
& \left[(1-\alpha)\left(\frac{1}{\beta}-(1-\delta)\right) + \frac{\theta(1-\sigma)}{\beta i_{ss}}\alpha\rho_y\right]\hat{k}_{t+1},
\end{aligned}
$$

while from (6.169) we get:

$$
\begin{aligned}
0 \quad = \quad & \frac{\sigma-\theta(1-\sigma)}{\beta}\hat{c}_t + \frac{\theta(1-\sigma)}{\beta i_{ss}}(1-\rho_i)\hat{\imath}_t - \frac{\sigma-\theta(1-\sigma)}{\beta}E_t\hat{c}_{t+1} + \\
& \left[\left(\frac{1}{\beta}-(1-\delta)\right) - \frac{\theta(1-\sigma)}{\beta i_{ss}}\rho_y\right]\rho_A\hat{A}_t - \qquad (6.174) \\
& \left[(1-\alpha)\left(\frac{1}{\beta}-(1-\delta)\right) + \frac{\theta(1-\sigma)}{\beta i_{ss}}\alpha\rho_y\right]\hat{k}_{t+1},
\end{aligned}
$$

Therefore, we can initially reduce the solution to the model to equations (6.163), (6.173) and (6.149), whose matrix representation is:

$$
\underbrace{\begin{bmatrix} 0 & k_{ss} & 0 \\ \frac{\sigma-\theta(1-\sigma)}{\beta} & \left[(1-\alpha)\left(\frac{1}{\beta}-(1-\delta)\right)+\frac{\theta(1-\sigma)}{\beta i_{ss}}\alpha\rho_y\right] & -\frac{\theta(1-\sigma)}{\beta i_{ss}}(1-\rho_i) \\ 0 & 0 & 1 \end{bmatrix}}_{D}\underbrace{\begin{bmatrix} E_t\hat{c}_{t+1} \\ \hat{k}_{t+1} \\ \hat{\imath}_t \end{bmatrix}}_{E_t v_{t+1}}
$$

$$
= \underbrace{\begin{bmatrix} -c_{ss} & \alpha A_{ss}k_{ss}^{\alpha}+(1-\delta)k_{ss} & 0 \\ \frac{\sigma-\theta(1-\sigma)}{\beta} & 0 & 0 \\ 0 & \alpha\rho_y & \rho_i \end{bmatrix}}_{G}\underbrace{\begin{bmatrix} \hat{c}_t \\ \hat{k}_t \\ \hat{\imath}_{t-1} \end{bmatrix}}_{v_t} +
$$

$$
\underbrace{\begin{bmatrix} A_{ss}k_{ss}^{\alpha} \\ \left(\frac{1}{\beta}-(1-\delta)\right)-\frac{\theta(1-\sigma)}{\beta i_{ss}}\rho_y \\ \rho_y \end{bmatrix}}_{H}\rho_A\hat{A}_t + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{J}\varepsilon_{i,t}
$$

that is,

$$DE_t v_{t+1} = G v_t + H \rho_A \hat{A}_t + J \varepsilon_{i,t},$$

or,

$$E_t v_{t+1} = \Gamma_1 v_t + \Gamma_2 \hat{A}_t + \Gamma_3 \varepsilon_{i,t}, \tag{6.175}$$

where $\Gamma_1 = D^{-1}G$, $\Gamma_2 = D^{-1}H\rho_A$, $\Gamma_3 = D^{-1}J$. Matrix Γ_1 is 3×3, with two stable and one unstable eigenvalues, whose associated eigenvector allows us to compute the value of the control variable (consumption) as a function of the two states $(\left\{\hat{k}_t, \hat{\imath}_{t-1}\right\})$.

As in previous model economies, we now apply Blanchard and Kahn's approach to obtain the numerical solution: Let $\Gamma_1 = M\Lambda M^{-1}$, where Λ and M are the matrices of eigenvalues and eigenvectors, respectively. Without loss of generality, let us assume that $|\lambda_1|, |\lambda_2| < 1$, $|\lambda_3| > 1$. Then, expression (6.175) will be equivalent to the system:

$$m_{11}E_t\hat{c}_{t+1}+m_{12}\hat{k}_{t+1}+m_{13}\hat{\imath}_t = \lambda_1\left(m_{11}\hat{c}_t + m_{12}\hat{k}_t + m_{13}\hat{\imath}_{t-1}\right)+L_1\hat{A}_t+Q_1\varepsilon_{i,t} \tag{6.176}$$

$$m_{21}E_t\hat{c}_{t+1}+m_{22}\hat{k}_{t+1}+m_{23}\hat{\imath}_t = \lambda_2\left(m_{21}\hat{c}_t + m_{22}\hat{k}_t + m_{23}\hat{\imath}_{t-1}\right)+L_2\hat{A}_t+Q_2\varepsilon_{i,t} \tag{6.177}$$

$$m_{31}E_t\hat{c}_{t+1}+m_{32}\hat{k}_{t+1}+m_{33}\hat{\imath}_t = \lambda_3\left(m_{31}\hat{c}_t + m_{32}\hat{k}_t + m_{33}\hat{\imath}_{t-1}\right)+L_3\hat{A}_t+Q_3\varepsilon_{i,t} \tag{6.178}$$

where $L = (L_1, L_2, L_3)' = M^{-1}\Gamma_2$, $Q = (Q_1, Q_2, Q_3)' = M^{-1}\Gamma_3$. Expression (6.178) can be solved forwards applying the law of iterated expectations. Indeed, let $z_t = m_{31}\hat{c}_t + m_{32}\hat{k}_t + m_{33}\hat{\imath}_{t-1}$. Then,

$$z_t = \frac{L_3}{\rho_A - \lambda_3}\hat{A}_t - \frac{Q_3}{\lambda_3}\varepsilon_{i,t},$$

that is,

$$\hat{c}_t = -\frac{m_{32}}{m_{31}}\hat{k}_t - \frac{m_{33}}{m_{31}}\hat{\imath}_{t-1} + \frac{L_3/m_{31}}{\rho_A - \lambda_3}\hat{A}_t - \frac{Q_3}{\lambda_3 m_{31}}\varepsilon_{i,t}, \tag{6.179}$$

which is the stability condition for the system in differences (6.175), that determines consumption as a function of the two state variables.

Plugging this stability condition in (6.176) and (6.177), we can write the two state variables as functions of their own past:

$$
\begin{aligned}
& \left(m_{12} - \frac{m_{11}m_{32}}{m_{31}}\right)\hat{k}_{t+1} + \left(m_{13} - \frac{m_{11}m_{33}}{m_{31}}\right)\hat{\imath}_t \\
= \quad & \lambda_1 \left(m_{12} - \frac{m_{11}m_{32}}{m_{31}}\right)\hat{k}_t + \lambda_1 \left(m_{13} - \frac{m_{11}m_{33}}{m_{31}}\right)\hat{\imath}_{t-1} + \qquad (6.180) \\
& + \left(L_1 + \frac{L_3 m_{11}/m_{31}}{\rho_A - \lambda_3}(\lambda_1 - \rho_A)\right)\hat{A}_t + \left(Q_1 - \frac{Q_3 m_{11}\lambda_1}{\lambda_3 m_{31}}\right)\varepsilon_{i,t} \qquad (6.181)
\end{aligned}
$$

$$
\begin{aligned}
& \left(m_{22} - \frac{m_{21}m_{32}}{m_{31}}\right)\hat{k}_{t+1} + \left(m_{23} - \frac{m_{21}m_{33}}{m_{31}}\right)\hat{\imath}_t \\
= \quad & \lambda_2 \left(m_{22} - \frac{m_{21}m_{32}}{m_{31}}\right)\hat{k}_t + \lambda_2 \left(m_{23} - \frac{m_{21}m_{33}}{m_{31}}\right)\hat{\imath}_{t-1} + \\
& + \left(L_2 + \frac{L_3 m_{21}/m_{31}}{\rho_A - \lambda_3}(\lambda_2 - \rho_A)\right)\hat{A}_t + \left(Q_2 - \frac{Q_3 m_{21}\lambda_2}{\lambda_3 m_{31}}\right)\varepsilon_{i,t} \qquad (6.182)
\end{aligned}
$$

or, in matrix form:

$$
\begin{bmatrix} m_{12}-\frac{m_{11}m_{32}}{m_{31}} & m_{13}-\frac{m_{11}m_{33}}{m_{31}} & -\left(L_1+\frac{L_3 m_{11}/m_{31}}{\rho_A-\lambda_3}(\lambda_1-\rho_A)\right) \\ m_{22}-\frac{m_{21}m_{32}}{m_{31}} & m_{23}-\frac{m_{21}m_{33}}{m_{31}} & -\left(L_2+\frac{L_3 m_{21}/m_{31}}{\rho_A-\lambda_3}(\lambda_2-\rho_A)\right) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{k}_{t+1} \\ \hat{\imath}_t \\ \hat{A}_t \end{bmatrix}
$$

$$
= \begin{bmatrix} \lambda_1\left(m_{12}-\frac{m_{11}m_{32}}{m_{31}}\right) & \lambda_1\left(m_{13}-\frac{m_{11}m_{33}}{m_{31}}\right) & 0 \\ \lambda_2\left(m_{22}-\frac{m_{21}m_{32}}{m_{31}}\right) & \lambda_2\left(m_{23}-\frac{m_{21}m_{33}}{m_{31}}\right) & 0 \\ 0 & 0 & \rho_A \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ \hat{\imath}_{t-1} \\ \hat{A}_{t-1} \end{bmatrix} +
$$

$$
\begin{bmatrix} Q_1-\frac{Q_3 m_{11}\lambda_1}{\lambda_3 m_{31}} & 0 \\ Q_2-\frac{Q_3 m_{21}\lambda_2}{\lambda_3 m_{31}} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{i,t} \\ \varepsilon_{A,t} \end{bmatrix}. \tag{6.183}
$$